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Title: Precision sensing assisted by quantum-classical computation

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Precision sensing assisted by quantum-classical computation

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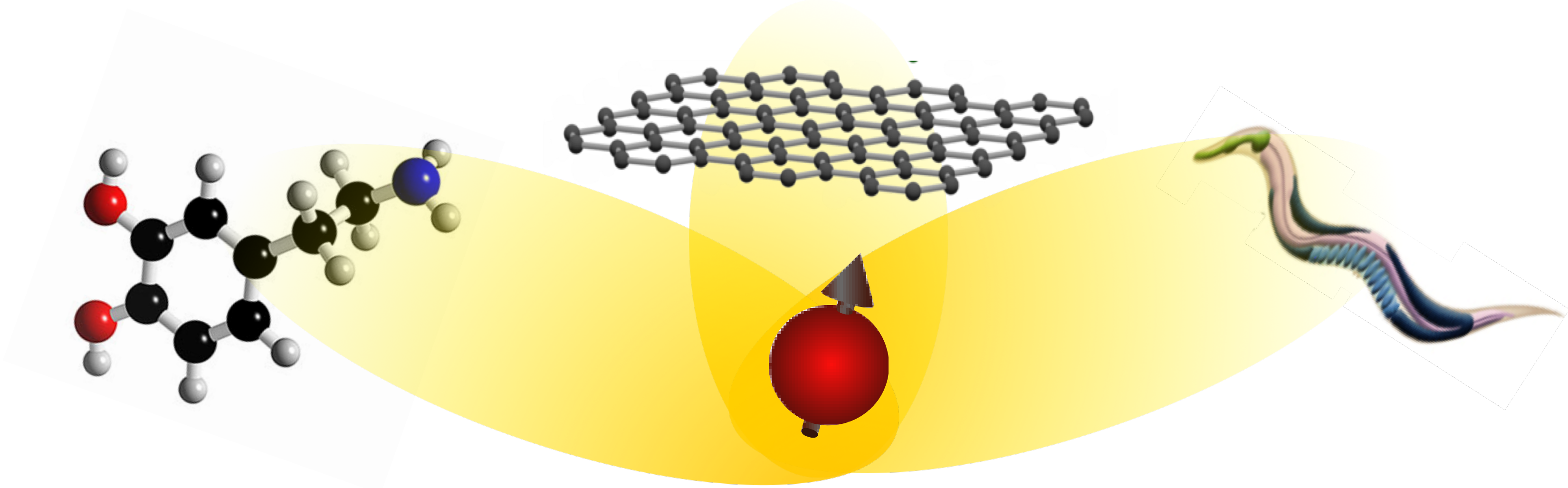
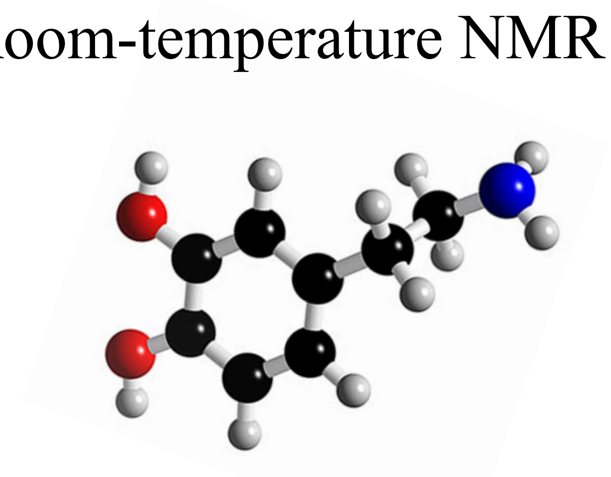
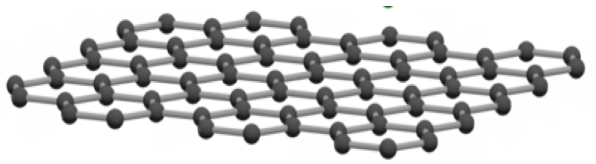


Quantum sensing for…

Room-temperature NMR



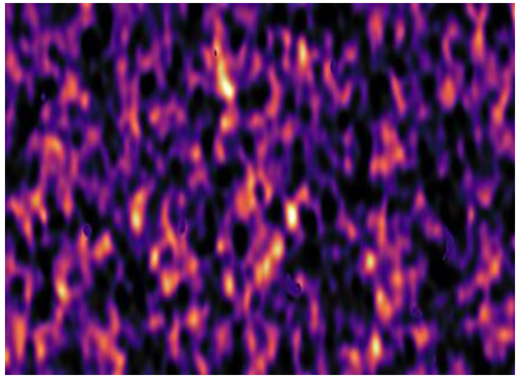
Magnetic material



Living system
(e.g. C. elegance)

Gravitational wave detection

Dark matter

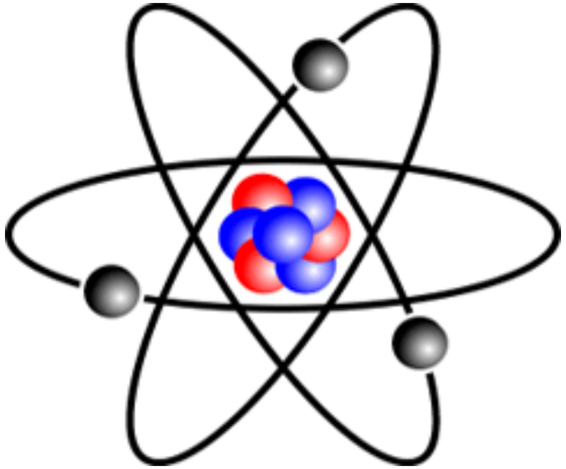


Chemistry, material science, medical science, cosmology…

Practical scenario
(not precise, but still interesting…)

Example of practical scenario: Molecular structure determination

atoms, protons, neutrons, electrons…



Schrödinger equation

$$i \boxed{\hbar} \frac{\partial}{\partial t} \boxed{\psi} = \boxed{H} \psi$$

Erwin Schrödinger

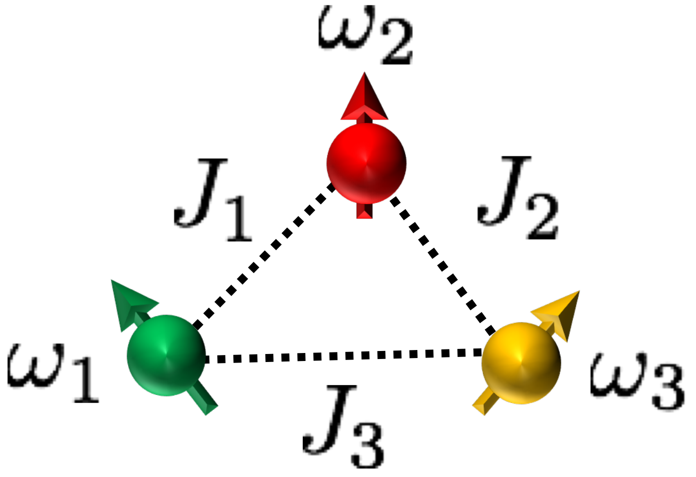
Planck constant (indicates a small world)

Wavefunction (state of the system)

Hamiltonian (gives total energy)

The information of molecule is included in the Hamiltonian

Example of practical scenario: Molecular structure determination



Number of qubits

Coupling / Graph

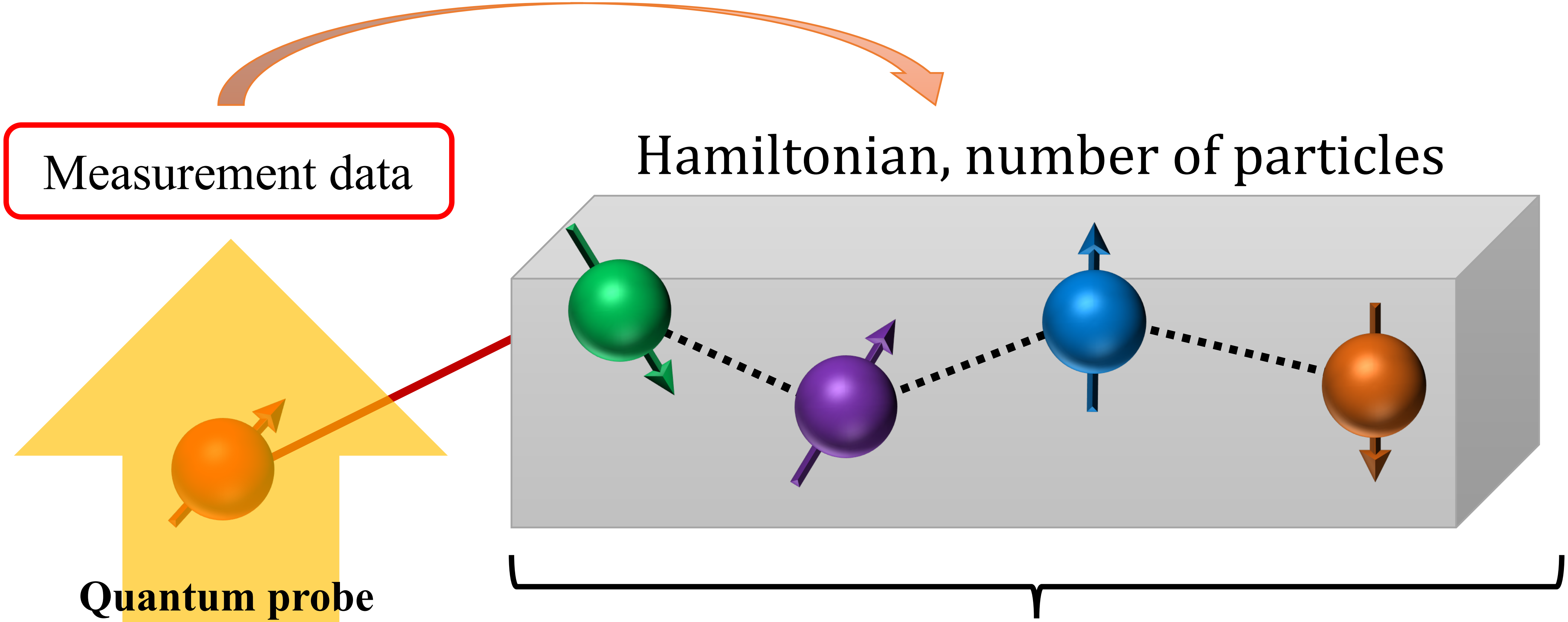
$$H = \sum_{k=1}^{N} \omega_k Z_k + \sum_{k \neq l} J_{kl} X_k X_l$$

Single-particle energy

Coupling strength

We want to extract this information from measurements

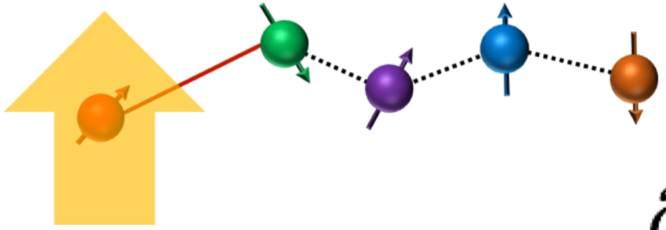
Example of practical scenario: Molecular structure determination



Unknown system: **(1) No direct measurement…**
(2) No direct control…

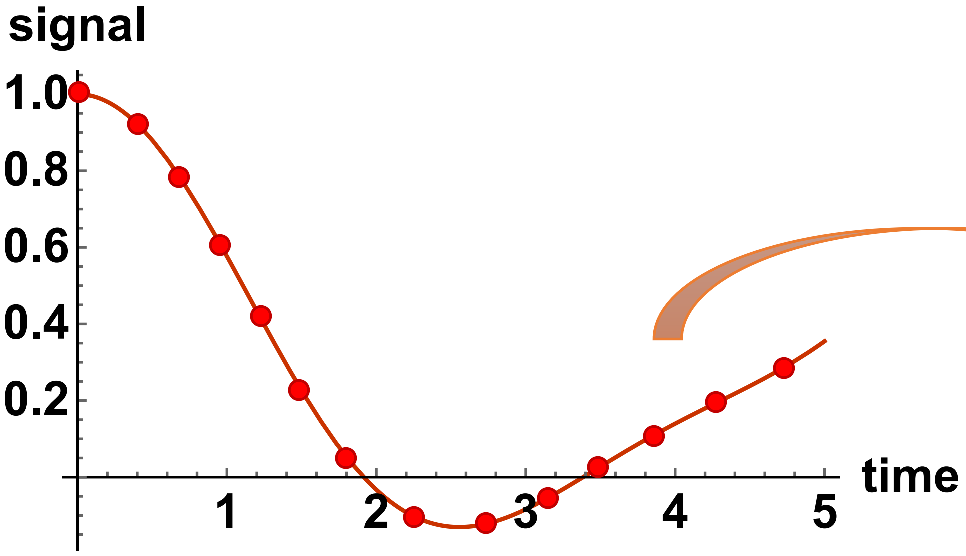
Estimation of Hamiltonian parameters:

Linearity of the quantum mechanical dynamics



Theoretical model: $$i\frac{\partial}{\partial t}\psi = H\psi$$

Experimental data:



Transfer function

$$\Xi(s)$$

Equivalent!

$$\Xi_{\mathrm{est}}(s)$$

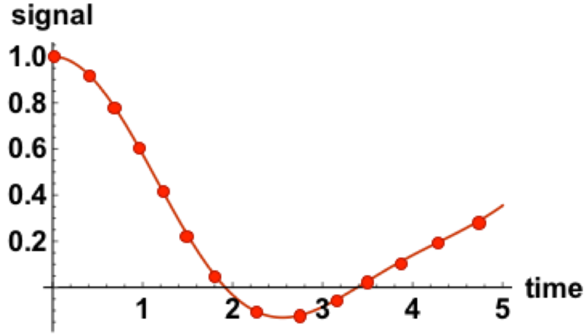
Estimation of Hamiltonian parameters:

Theoretical model: $i\frac{\partial}{\partial t}\psi = H\psi$

polynomials

$$\Xi(s) = \frac{s^m + g_2(\theta_1, \cdots, \theta_p)s^{m-1} + \cdots}{s^n + g_1(\theta_1, \cdots, \theta_p)s^{n-1} + \cdots}$$

Experimental data:



numbers

$$\Xi_{\mathrm{est}}(s) = \frac{s^m + a_2 s^{m-1} + \cdots}{s^n + a_1 s^{n-1} + \cdots}$$

equal

Estimation of Hamiltonian parameters:

We must have:

$$\Xi(s) = \Xi_{\text{est}}(s)$$

Coefficients of the Laplace variable *s* in $\Xi(s)$ are **polynomials.**
Coefficients of the Laplace variable *s* in $\Xi_{\text{est}}(s)$ are **numbers.**

System of polynomial equations

$$\begin{cases} f_1(\theta_1, \cdots, \theta_p) = 0 \\ f_2(\theta_1, \cdots, \theta_p) = 0 \\ f_3(\theta_1, \cdots, \theta_p) = 0 \\ \quad \vdots \\ f_q(\theta_1, \cdots, \theta_p) = 0 \end{cases}$$

Solve the equation directly

Find the parameters

$$\{\theta_1, \theta_2, \cdots, \theta_p\}$$

Precision sensing

Quantify ultimate precision

Quantum Fisher information (QFI):

$$I(\theta;\rho_\theta) = 8 \lim_{\epsilon\to 0} \frac{1-\mathbb{F}[\rho_\theta,\rho_{\theta+\epsilon}]}{\epsilon^2}$$

error

Fidelity: distance measure of two quantum states

Quantum Cramer-Rao bound:

$$\delta\theta^2_{\{\Pi_j\}} \geq \frac{1}{I(\theta;\rho_\theta)}$$

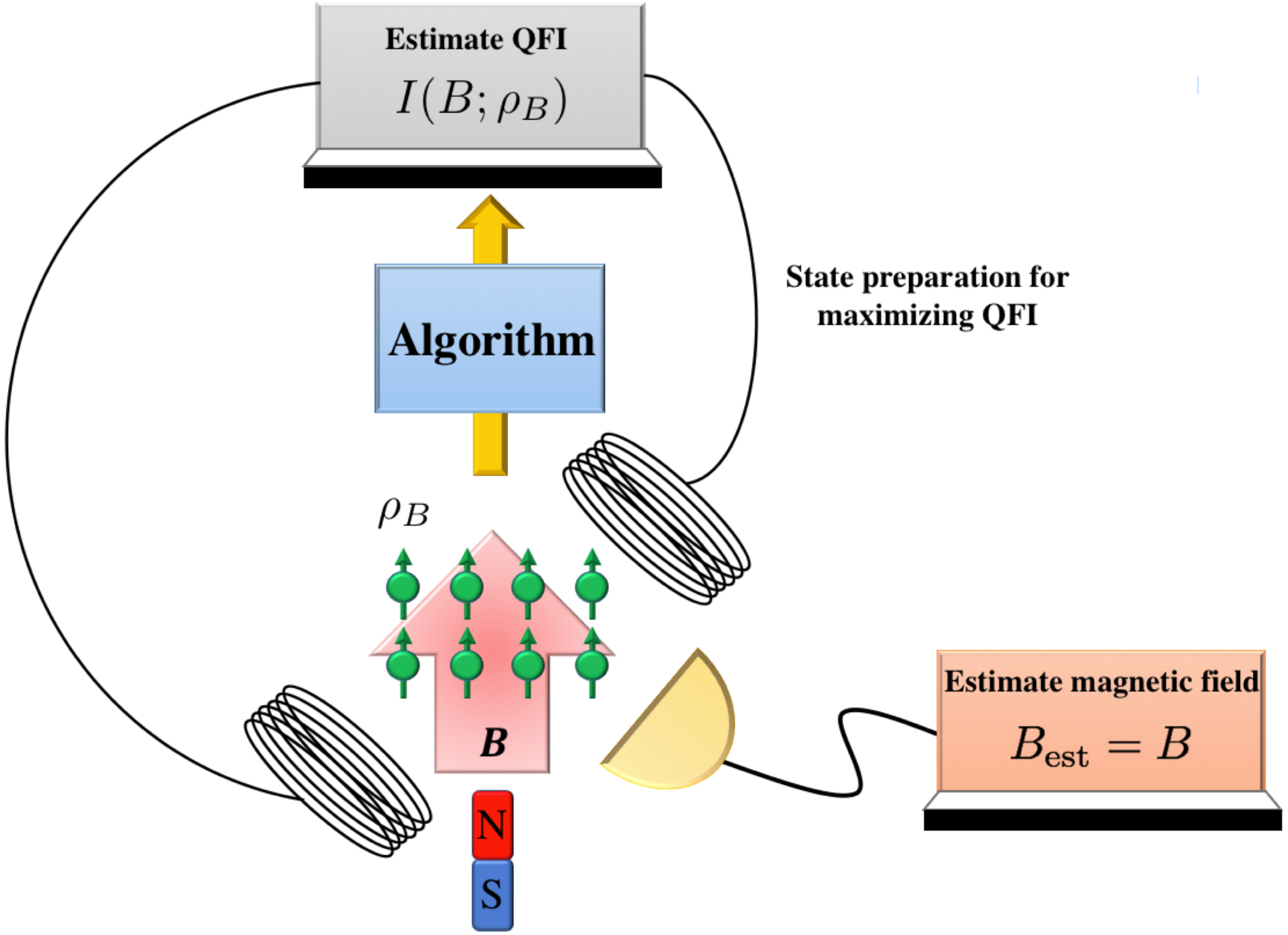
Ultimate precision limit (how good the quantum sensor is)

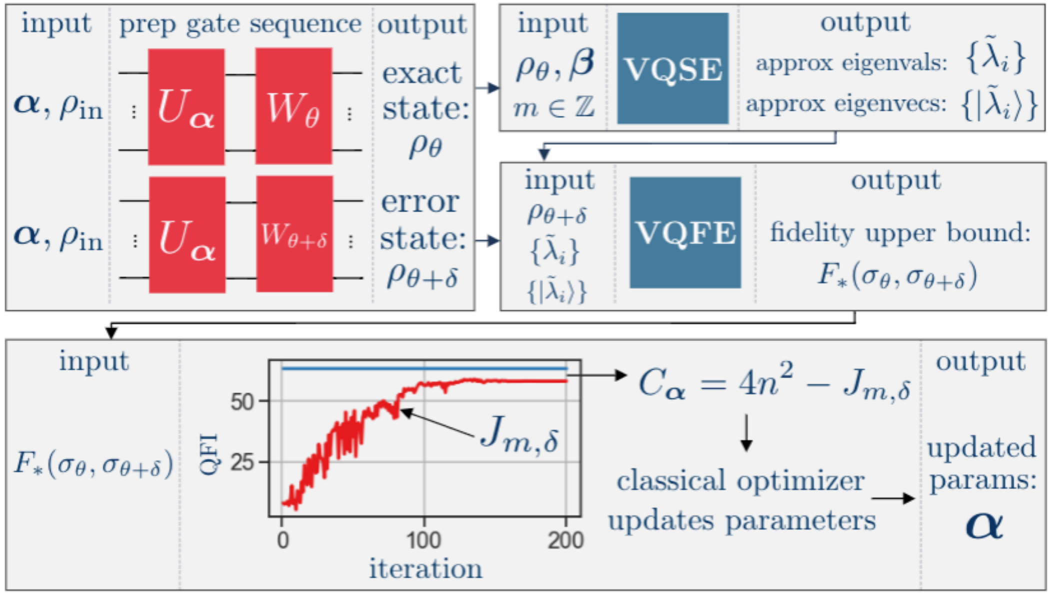
$\{\Pi_j\}$: set of measurements

For the optimal measurement $\{\Pi_j^*\}$, the variance satisfies the bound

$$\delta\theta^2_{\{\Pi_j^*\}} = \delta\theta_{\min} = \frac{1}{I(\theta;\rho_\theta)}$$

Quantum sensing assisted by a quantum-classical computer





1. Computing the lower bound of QFI by using the truncated state constructed with smaller number of eigenvalues

2. Feedback control to prepare the good state

3. Finding optimal measurement

Quantum sensing assisted by a quantum-classical computer

As increase the size of truncated state, the lower bound is getting closer to the real QFI

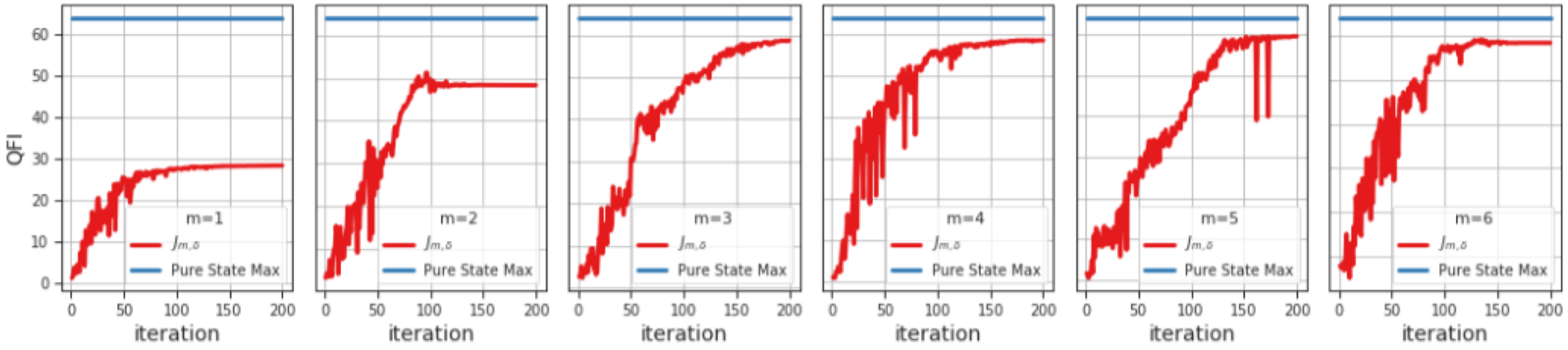


FIG. 3. Figure showing lower bound, $J_{m,\delta}$, versus iteration for different m values (number of non-zero eigenvalues kept). A 4-qubit state of purity ~ 0.95 was used to generate this data. The key point is that our lower bound increases with m.

Ongoing research: Finding optimal measurement